Euclidean properties of conics

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Abstract

This course delves into the Euclidean properties of conics, and display their usefulness in olympiad geometry. Conics, encompassing circles, ellipses, parabolas, and hyperbolas, have been integral to classical geometry and have been widely studied. Our investigation involves an in-depth analysis of the fundamental relation between conics and (isogonal) conjugation with respect to a triangle. Throughout the course, we will be using and applying properties of the cross-ratio, isogonal conjugation in a triangle, pole-polar transformation, Steiner's definition of a conic, Desargues involution theorem, and Poncelet's porism. At last, we will apply the theory to challenging olympiad geometry problems from various contests such as the IMO, USA TSTST, and AOPS contests like SAGF 2024.

Prerequisites: Projective geometry. Dececently familiar with cross-ratio.

Note: If not enough participants are familiar with the prerequisities, we might do them as well.