Definition 1. The affine span is $\text{aff}\{v_0, \ldots, v_n\} = \{v_0 + u \mid u \in \text{span}\{v_1 - v_0, \ldots, v_n - v_0\}\}$

Problem 1. $\text{aff}\{v_0, \ldots, v_n\} = \{\beta_0 v_0 + \cdots + \beta_n v_n \mid \beta_i \in \mathbb{R},\ \beta_1 + \cdots + \beta_n = 1\}$

Definition 2. $\{v_0, \ldots, v_n\}$ are *affine independent* if no v_i is in the affine span of the others.

Problem 2. If $\{v_0, \ldots, v_n\}$ are affine independent, then there is a linearly independent subset of size n.

Definition 3. The functions $T_1, \ldots, T_n : \mathbb{R} \to \mathbb{R}$ are $\mathcal{P}$*-affine independent* if

$$\lambda_1 T_1 + \cdots + \lambda_n T_n = c \implies \lambda_1 = \cdots = \lambda_n = 0,$$

where c is some constant function.

Problem 3. $\mathcal{P}$-affine independent $\implies$ linearly independent $\implies$ affine independent.

Problem 4. If $T_1, \ldots, T_n$ are linearly independent, then there is a subset of size $n - 1$ that is $\mathcal{P}$-affine independent.

Problem 5. If $T_1, \ldots, T_n$ are linearly independent but not $\mathcal{P}$-affine independent, then there is a unique smallest subset that is not $\mathcal{P}$-affine independent.

Problem 6. Let $T_1, \ldots, T_n : \mathbb{R} \to \mathbb{R}$, and

$$T(x) := \begin{pmatrix} T_1(x) \\ \vdots \\ T_n(x) \end{pmatrix}.$$

Then $T_1, \ldots, T_n$ are $\mathcal{P}$-affine independent iff $\exists x_0, \ldots, x_n$ s.t. the points $T(x_0), \ldots, T(x_n)$ are affine independent. Moreover, this is equivalent to that

$$\begin{pmatrix} T_k(x_1) - T_k(x_0) \\ T_k(x_2) - T_k(x_0) \\ \vdots \\ T_k(x_n) - T_k(x_0) \end{pmatrix},$$

$k \in \{0, \ldots, n\}$, are linearly independent.

Problem 7. Suppose T_1, T_2 are $\mathcal{P}$-affine independent, and ζ_1, ζ_2 are $\mathcal{P}$-affine independent. Then there are no functions s.t $T_1(x)\zeta_1(\theta) + T_2(x)\zeta_2(\theta) = f(x)g(\theta) + f_2(x) + g_2(\theta)$.

Also generalized version.