The Brauer group and around

The Brauer group of a field k is a way to organize certain kinds of algebraic objects called central simple algebras into abelian groups. These ideas first came up when mathematicians were trying to classify division algebras, but they also turn out to have exciting connections to geometry. For example, the Brauer group helps us understand projective varieties over k that look like projective space when viewed over the algebraic closure of k. These varieties are called Severi–Brauer varieties.

In this mini-course, we will start by learning about quaternions, one of the simplest examples of central simple algebras. Quaternions are a great way to build intuition for more complicated examples. Then, we'll look at cyclic algebras, which are like a generalization of quaternions. After that, we'll explore the broader theory of Brauer groups and their structure. If we have time, we'll also discuss how these ideas connect to geometry, focusing on Severi–Brauer varieties.

This course will focus on solving problems and talking through the ideas together. I hope this will make the topics easier to understand and more enjoyable. I look forward to learning together with all of you!

Prerequisites: This course will require some basic knowledge of linear algebra, such as fields, matrices, and related things. You can learn these topics in Alex Villaro Krüger's course, "Introduction to Vector Spaces". Familiarity with basic algebra would be helpful, but it is not strictly necessary!