

Ehrhart theory

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Abstract

Let $P \subset \mathbb{R}^d$ be a polytope whose vertices have integer coordinates. Scaling this polytope by positive integers gives polytopes $P, 2P, 3P, \dots$ and we wish to count the number of points with integer coordinates lying in nP . Ehrhart proved that this is always a polynomial in n , and it is therefore called the Ehrhart polynomial of P .

In the first lecture, we will see how to prove this theorem. The proof uses ideas from geometry, combinatorics, and algebra, and we will discover and work with some of these ideas through solving problems leading up to the main result. The second lecture will be more open, and through problems many different further results may be explored, (probably) including Stanley's nonnegativity theorem, the Ehrhart polynomial for order polytopes, and Ehrhart-Macdonald reciprocity. If some topic seems interesting, we can cover it together on the board.