Obstructions to rational points: the Hasse principle and Brauer groups

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Abstract

We will start by learning how to find integer solutions of equations. In general this is a very hard question, but for some equations the Hasse principle applies! This is roughly the idea that if we find a solution of the equation modulo p^n for every prime power, then we can also find a solution of the equation with integers. At first this seems like a lot of work, but it turns out that this procedure is surprisingly simple to carry out.

However, sometimes the Hasse principle fails! The rest of the time will be occupied by trying to understand how and why. This leads to the so-called Brauer-Manin obstruction, which I hope to motivate and explain through explicit examples. For this part it is especially encouraged to also follow Sasha's course on Brauer groups!