

Heights and valuations

Magnus Ridder Olsen

January 2025

In this series of exercise sessions, we will explore the arithmetic complexity of a point called height. This concept is fundamental in the field of arithmetic geometry, where it lets employ the powerful method of Fermat descend in a situation where there is no apparent notion of size.

The one-dimensional case is essentially the study of valuations of global fields and how these behave under extensions. This part will be accessible to anyone with an understanding of abstract algebra. It will also be possible to digress and discuss local fields if this is of interest.

To discuss the higher-dimensional case we will presuppose a rudimentary understanding of projective varieties. Furthermore, black boxing a few theorems about abelian varieties will make it possible to discuss the Néron–Tate height. This is a positive definite quadratic form that is essential in the study of the arithmetic of abelian varieties.

If time allows we can also discuss arithmetic dynamics