

INTRODUCTION TO VECTOR SPACES

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I will prepare a problem sheet for the class. By solving the problems, you will basically understand what a vector space is, and you will understand the idea of "linear independence".

PROBLEM 1. A 4×4 table can have a "+" or a "-" sign in each of the 16 cells. To begin with, all cells contain only "+". It is possible to perform an operation, where you choose a cell, and change the sign in this cell and in all the neighbouring cells. How many different positions can you obtain, by continuing to apply this operation?

PROBLEM 2 (КВАНТ 1972). An $n \times m$ table contains rational numbers such that for any two rows and any two columns, we obtain a rectangle where the sum of numbers in opposite corners are equal. Some of the numbers in the table have been erased. Prove that there are at least $(n+m-1)$ numbers left.

IDEA. How can the space of tables that satisfy the condition be considered as a vector space? How is the dimensional of this vector space related to the minimal number of fields required to restore the table?