(D)DIT

Problem 1 (Isogonal Conjugate Lemma)**.** Let ABC be a triangle and let P and Q be points such that AP and AQ are isogonal with respect to $\angle BAC$. Let $X = BP \cap CQ$ and $Y = CP \cap BQ$. Prove that AX and AY are isogonal with respect to $\angle BAC$.

Problem 2 (USAMO 2012 P5)**.** Let P be a point and ABC a triangle. l is a line through P. The reflections of PA, PB, and PC in l intersect BC, CA, and AB at A', B', and C'. Prove that A', B', and C' are collinear.

Problem 3 (Orthotransversal)**.** Let ABC be a triangle and P a point. X, Y, and Z are on BC, CA, and AB such that $\angle XAP = \angle YBP = \angle ZCP = 90°$. Prove that X, Y, and Z are collinear.

Problem 4 (Ping-pong Lemma)**.** Let $\mathcal{C}$ be a conic section and X a point on $\mathcal{C}$. P, Q, and R are three points on a line. PX intersects $\mathcal{C}$ at Y. QY intersects $\mathcal{C}$ at Z. RZ intersects $\mathcal{C}$ at W. XW intersects $\overline{PQR}$ at S. Prove that S is fixed as X varies on $\mathcal{C}$.

Problem 5 (Well-known)**.** Let ABC be a triangle and let D be the point where the incircle touches the side BC. D' is the point where the A-excircle touches BC. Prove that $BD = D'C$.

Problem 6 (USA)**.** Let H be the orthocenter in $\triangle ABC$. Ω is the circumcircle of $\triangle ABC$ with center O. The midpoints of BC, CA, and AB are M_A, M_B, and M_C. The lines AM_A, BM_B, and CM_C intersect Ω again at P_A, P_B, and P_C. The lines HM_A, HM_B, and HM_C are extended beyond H and intersect Ω again at Q_A, Q_B, and Q_C. Prove that P_AQ_A, P_BQ_B, and P_CQ_C intersect on OH.

Problem 7 (AOPS)**.** Let Γ be the circumcircle of $\triangle ABC$. A circle is internally tangent to AB, AC, and Γ at P, Q, and R. Let $S = AR \cap PQ$. Prove that $\angle PSB = \angle QSC$.

Problem 8 (AOPS)**.** Let ABC be a right-angled triangle with hypotenuse BC. D and E are on AC and AB and $F = CE \cap BD$. The circumcircles of $\triangle ABC$ and $\triangle ADE$ intersect at $G \neq A$. Prove that $AG \perp GF$.

Problem 9 (Geometry in Figures)**.** Let $\mathcal{H}$ be a hyperbola. A line l intersects $\mathcal{H}$ at A and B and the asymptotes of the hyperbola at C and D. Prove that $AC = BD$.

Problem 10 (CSP 2018 Day 2)**.** Let ABC be a triangle with circumcenter O. P and Q are the centers of (OAB) and (OAC) respectively. R is the reflection of O in BC. $X = RB \cap CP$, $Y = RC \cap BQ$. Prove that $\angle BAX = \angle CAY$.

Problem 11 (IMOSL 2007 G3)**.** The diagonals of a trapezoid $ABCD$ ($BC \parallel AD$) intersect at P. Q lies between BC and AD such that $\angle AQD = \angle CQB$ and such that the line CD separates P and Q. Prove that $\angle BQP = \angle DAQ$.

Problem 12 (Azerbaijan IMO TST 2022 Day 1 P3, generalized)**.** Let ABC be a triangle with circumcircle Γ. D is a fixed point on Γ and P is a variable point on AD. X and Y are on AC and AB respectively such that the reflections of B and C in $\overline{AD}$ lie on XP and YP respectively. The circumcircle of $\triangle AXY$ intersects Γ at $Z \neq A$. Prove that $\angle PZD$ is equal to the angle formed by the lines $\overline{AD}$ and $\overline{BC}$.

Problem 13 (Iran RMM TST 2019 Day 2 P6)**.** Let $ABCD$ be a cyclic quadrilateral with circumcircle ω. M is a point on ω. $E = AB \cap CD$ and $F = AD \cap BC$. ME intersects AD and BC at P and Q. MF intersects AB and CD at R and S. The lines PS and RQ intersect at X. Prove that MX passes through a fixed point as M varies on ω.

Problem 14 (All-Russian MO 2015 P7). Let ABC be a triangle. H is the foot from A on BC and M is the midpoint of BC. P and Q are on AC and AB such that $PM \perp AB$ and $QM \perp AC$. The circumcircle of $\triangle PMQ$ intersects BC at $X \neq M$. Prove that $BH = CX$.

Problem 15 (Serbia 2017 P6). Let ABC be a triangle and let the common external tangents to the circumcircle of $\triangle ABC$ and the A-excircle intersect BC at P and Q. Prove that $\angle PAB = \angle CAQ$.

Problem 16 (AOPS). Let ABC be a triangle with circumcircle Ω. M is the intersection of the A-angle bisector and Ω. D is on Ω such that $AD \parallel BC$. P is a variable point on AM. BP and CP intersect CA and AB at E and F. PD intersects Ω again at K. T is the intersection of KM and EF. Prove that AT is tangent to Ω.

Problem 17 (Extension of IMOSL 2011 G4). Let ABC be a triangle with circumcircle Γ and let D, E, and F be the midpoints of BC, CA, and AB. The circle ω passes through E and F and is tangent to Γ at $T \neq A$. The line AT intersects ω at $R \neq T$. Show that T, R, D, and the center of Γ are concyclic.

Problem 18 (IMOSL 2015 G7). Let $ABCD$ be a convex quadrilateral and let P, Q, R, and S be points on AB, BC, CD, and DA. The lines PR and QS intersect at O. Assume $APOS$, $BQOP$, $CROQ$, and $DSOR$ each possess an incircle. Prove that AC, PQ, and RS are concurrent.

Problem 19 (ELMO 2016 P2). Let ABC be a triangle with circumcircle Γ. The tangents to Γ at B and C intersect at D. B' is the reflection of B in CA and C' is the reflection of C in AB. O is the circumcenter of $\triangle DB'C'$. Prove that $AO \perp BC$.

Problem 20 (Taiwan TST 2014 Day 3 P3). Let ABC be a triangle with circumcircle Γ. Let M be a point on Γ. The tangents from M to the incircle of $\triangle ABC$ intersect BC at X and Y. Prove that the circumcircle of $\triangle MXY$ passes through a fixed point as M varies on Γ.

Problem 21 (IMOSL 2012 G8). Let O be the circumcenter of $\triangle ABC$ and l an arbitrary line. The foot from O on l is P and the lines BC, CA, and AB intersect l at X, Y, and Z. Prove that the circumcircles of $\triangle AXP$, $\triangle BYP$, and $\triangle CZP$ are coaxial.

Problem 22 (ELMOSL 2018 G4). Let $ABCDEF$ be a cyclic hexagon with circumcircle Ω and such that the orthocenters of $\triangle ACE$ and $\triangle BDF$ are coincident. Assume that CE intersects BD and DF at X and Y. Prove that the circumcircle of $\triangle DXY$ and the line through A perpendicular to CE intersect on Ω.

Problem 23 (Geometry marathon P193). Let ABC be a triangle inscribed in a conic $\mathcal{C}$ (say ellipse for simplicity) such that its incenter lie on the line l joining the (real) foci of conic. l intersects $\mathcal{C}$ at R and S. AI meet the ellipse again at L. BC intersects LR and LS at X and Y.

Prove that $IX \perp IY$.

Problem 24 (Geometry marathon P158). Let U, V be two points lying on the circumcircle of $\triangle ABC$ such that UV passes through X_{22} of $\triangle ABC$.

Prove that the bicevian conic of U, V WRT $\triangle ABC$ is a rectangular hyperbola.

Problem 25 (Conjecture). Let $X \in \mathrm{Sh}(\mathrm{Man}_{\mathbb{Q}_p})$ be a $\mathbb{Q}_p$-analytic smooth Artin stack. Is the associated light condensed anima to X, étale locally, a profinite anima?