

6-Functor Formalisms and Étale Cohomology

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A 6-functor formalism provides a powerful framework for unifying the core structural properties of a cohomology theory into six functors. For decades, this formalism remained a heuristic principle—a “you know it when you see it” phenomenon—rather than a strictly defined mathematical object. However, recent developments in ∞ -categorical geometry have finally provided a rigorous foundation for these structures. This lecture aims to bridge that gap by providing a formal definition of a 6-functor formalism without assuming a background in ∞ -categories.

To ground these abstract concepts, we will also give an introduction to **étale cohomology of schemes** and prove it gives a 6 functor formalism.. Notably used by Deligne to resolve the Weil conjectures.

Background & Prerequisites To get the most out of this talk, participants should have a robust command of **category theory**, with some exposure to **homological algebra** and **derived categories**. The discussion on étale cohomology will assume a standard background in **algebraic geometry** (schemes and morphisms).

Familiarity with the following “nice-to-haves” will provide helpful context:

- Singular cohomology and sheaves on sites.
- Basics of ∞ -categories.
- Group cohomology.