(P,ω)-partitions

Mikael Häglund

Abstract

A partition of a positive integer n into k parts is a sequence of integers $\lambda_1, \dots, \lambda_k$ satisfying

$$\lambda_1 \geq \cdots \geq \lambda_k > 0$$
$$\lambda_1 + \cdots + \lambda_k = n.$$

Counting partitions is an interesting and well-studied problem. As an easy example of a result about counting partitions, let $a_k(n)$ denote the number of partitions of n into at most k parts. We then have

$$\sum_{n \geq 0} a_k(n) x^n = \frac{1}{(1-x)(1-x^2)\dots(1-x^k)}.$$

In the definition of integer partitions, we require a total order on the parts. Replacing this with an arbitrary partial order from a partially ordered set P gives so-called P-partitions, and selecting also some of the inequalities to be strict, we obtain the definition of (P,ω)-partitions. As always in combinatorics, we wish to count these things, to get for example results like the formula for the generating function above (we will indeed see such a generalization).

The first class will consist mostly of problem-solving in and around the theory of (P,ω)-partitions. Some of the topics in the problems are not directly related to (P,ω)-partitions, but are instead things that show up all the time in combinatorics and that I think are nice things to know about. Some of the results in the problems will also be used in the next class, where I will probably go through the main results in the theory of (P,ω)-partitions in more of a lecture.

There are no real prerequisites for this, but if you really want to prepare, try reading through chapter 1.4 of Enumerative Combinatorics Vol. 1 by Stanley, or even chapter 3.15, which is the part on (P,ω)-partitions.