# Additive combinatorics and Fourier Analysis in the setting of finite fields

B

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## 1 Abstract

How large can a subset $A$ of $(\mathbb{Z}/p\mathbb{Z})^n$ with no $x_1, x_2, x_3$ solving $x_1 + x_2 = x_3$ get and what does a maximal set like this look like? What about a subset with no 3-term arithmetic progressions, that is distinct $x_1, x_2, x_3$ solving $x_1 + x_3 = 2x_2$?

Even if they seem similar, the two questions turn out to be fundamentally different. The first condition, that can be reformulated to $A + A \ \cap \ A \ = \varnothing$ using the sumset notation, turns out to be rather easy: the solution is a product of the middle third of $\mathbb{Z}/p\mathbb{Z}$ with a hyperplane - a very structured set. On the other hand, finding bounds for the size of a maximal set following the second condition is a hard topic in research in additive combinatorics.

Generally speaking, additive combinatorics investigates additive structure in sets. Additive structure in a set can be imposed, for example, by conditions such as small sumsets or avoidance of solutions to simple linear equation systems. We discover then that such restrictions impose the subset to be "structured" in a more canonical sense, as being close to cosets of subspaces.

Discrete Fourier transform finds applications in several questions in additive combinatorics, as it also, in some sense, measures structure. Very roughly speaking, any subset is either random or structured. Techniques using Fourier analysis typically detect that large sets imposed by conditions as 3-term arithmetic progression avoidance cannot be too random and give information about their structure, that then can be used for combinatorial argumentation.

The only recommended, but not necessary, prerequisite for this course is the knowledge of very basic Fourier analysis.