Prerequisites: know what is a triangle, geometric intuition of a vector could be helpful.

Example 1. Fixing points A and B, find the centres of mass of:
$(A,2),(B,3)$; $(A,-3),(B,5)$; $(A,-8),(B,5)$;
$(A,0),(B,5)$; $(A,2),(A,1)$; $(A,\alpha),(A,-\alpha)$.

Example 2. Find the centre of mass for the two triangle:
(A, 2), (B,1), (C,4); (A,3), (B,1), (C,-2).

Problem 3. Find the centre of mass of (A, 2), (B, -3), (C, 5).

Example 4. Find the COM of $(A,\alpha),(B,\beta),(C,-\beta)$.

Example 5. Prove that all three medians of a triangle are concurrent and find the ratio in which they intersect.

Example 6. Points M,N,E,F are the midpoints of the sides AB,BC,CD,DA of a convex quadrilateral $ABCD$; K and L are the midpoints of its diagonals AC and BD. Prove that ME,NF,KL are concurrect and bisect eachother.

Example 7. Point M lays on AB and satisfies $AM:MB=2:5$, point N is on BC s.t. $BN:NC=4:3$. AN intersects CM at K. Find the ratios $AK:KN$ and $CK:KM$.

Problem 8. On the sides of triangle ABC lay points E and F s.t. $CE=\frac{2}{3}CA$ and $CF=\frac{4}{5}CB$. Segments BE and AF intersect at M. Find $AM:AF$ and $BM:BE$.

Problem 9. On the sides of triangle ABC points C_1,A_1,B_1 are marked; they satisfy $AC_1:C_1B=2:5$, $BA_1:A_1C=1:2$, $CB_1:B_1A=4:1$. Segments CC_1 and A_1B_1 intersect at E. Find $CE:EC_1$ and $B_1E:EA_1$.

Example 10. On the diagonals AC and BD (resp.) of a convex quadrilateral $ABCD$ points M and N are marked, s.t. $AM=6MC$ and $BN=6ND$. Prove that the midpoints of AB,CD,MN are colinear.

Example 11. The medians AK and CF of the base ABC of a trangular pyramid $ABCD$ intersect at E. Points L and M lay on the edges AD and CD s.t. $CM=\frac{3}{4}CD$, $DL=\frac{1}{4}DA$. Prove that FM,KL,DE are concurrent and find the ratios at which they intersect.

Problem 12. On the sides AB and CD of a triangular pyramid $ABCD$, points P and Q are marked, s.t. $AP:PB=3:1$ and $CQ:QD=3:1$. Prove that the midpoints of PQ,AC,BD are colinear.

Problem 13. The base $ABCD$ of the pyramid $SABCD$ is a parallelogram. A plane intersects AS,BS,CS,DS at points A_1,B_1,C_1,D_1 respectively; furthermore $AA_1=2A_1S$, $BB_1=4B_1S$ and $CC_1=3C_1S$. Find $DD_1:D_1S$.

Example 14 (finally gets interesting). A triangle PRQ is given. On its sides PR,RQ,QP (resp.), points E,F,L are marked, satisfying $PE:ER=3:4$, $RF:FQ=3:2$ and $QL:LP=1:2$. Segments RF and EF intersect at K. Find $RK:KL$ and $EK:KF$.

Problem 15. On edges DA,DB,DC of a triangular pyramid $ABCD$, points M,N,L are marked s.t. $AM:MD=3:2$, $BN:ND=2:5$, $CL:LD=1:3$. Point E is the centroid of ABC. The plane containing M,N,L intersects DE at K. Find $DK:KE$.

Example 16. Pyramid $SABCD$ has parallelogram $ABCD$ as a base. On edges SA,SB,SC lay points A_1,B_1,C_1; according to ratios $SA_1=\frac{1}{3}SA$, $SB_1=\frac{1}{4}BS$, $SC_1=\frac{1}{5}SC$. Prove that the plane $(A_1B_1C_1)$ intersects SD and find the ratio.

Example 17. On edges DA,DB,DC of a triangular pyramid $ABCD$ are points M,N,K, such that $DM=\frac{1}{3}DA$, $DN=\frac{1}{4}DB$, $DK=\frac{3}{5}DC$. E is the centroid of ABC. The plane containing M,N,K intersects DE at P. Find $DP:PE$.

Problem 18. Points E_1,E_2,E_3,E_4,E_5,E_6 are the midpoints of sides AB,BC,CD,DE,EF,FA of a convex hexagon $ABCDEF$. Prove that the centroids of E_1,E_3,E_5 and E_2,E_4,E_6 coincide.

Problem 19. Triangle ABC satisfies $\angle A=30^\circ$, $\angle B=60^\circ$. The altitude CD and median AF intersect at E. Find $AE:EF$ and $CE:ED$.

Example 20. Prove that the angle bisectors of a triangle are concurrent; find at what ratio they intersect each other (in terms of the sidelengths).

Example 21. Side AB of triangle ABC is of length 3, AC of length 4. median BB_1 and angle bisector AA_1 intersect at G. Find the area of ABC if that of BGA_1 is S.

Problem 22. The sides of triangle ABC satisfy $AB=3, BC=2$. Point D divides side AB in ratio $AD:DB=1:2$. The angle bisector BE and segment CD intersect at F. Find the area of ABC, if the area of BCF is S.

Problem 23. An acute triangle ABC has angles of measure α,β,γ respectively. Prove that the altitudes are concurrect and express the ratios, in which they divide one another.