Problem Set 2: Representations of GL_n

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Problem 1. Find a representation of $\mathbb{Z}$ which is not completely reducible.

Problem 2. a) Show that all the irreducible representations of a commutative group are 1-dimensional.

b) Find all the irreducible representations of the group $(\mathbb{C}^*)^n$.

Problem 3. Prove that S^kV, Λ^kV are irreducible representations of $GL(V)$.

Problem 4. Find the weights of S^kV, Λ^kV. Write down the characters of those representations.

Problem 5. The aim of this problem is to describe the rules of the decomposition of tensor products of representations of GL_2.

a) Show that the representation $\left(\Lambda^2(\mathbb{C}^2)\right)^{\lambda_2} \otimes S^{\lambda_1-\lambda_2}(\mathbb{C}^2)$ is irreducible. Thus $V_{\lambda_1,\lambda_2} = \left(\Lambda^2(\mathbb{C}^2)\right)^{\lambda_2} \otimes S^{\lambda_1-\lambda_2}(\mathbb{C}^2)$

b) Write down the character of V_{λ_1,λ_2}

c) Using b), decompose $V_{\lambda_1,\lambda_2} \otimes V_{\mu_1,\mu_2}$

Problem 6. Decompose into irreducibles the following representations of $GL(V) = GL_3$:
a) $V \otimes V$, b) $\Lambda^2V \otimes V$, c) $S^2V \otimes V$